JR 2.9

2.9 Suppose there are only two goods and that a consumer's choice function $\mathbf{x}(\mathbf{p}, y)$ satisfies budget balancedness, $\mathbf{p} \cdot \mathbf{x}(\mathbf{p}, y) = y \; \forall \; (\mathbf{p}, y)$. Show the following:

(a) If $\mathbf{x}(\mathbf{p}, y)$ is homogeneous of degree zero in $(\mathbf{p}, y)$, then the Slutsky matrix associated with $\mathbf{x}(\mathbf{p}, y)$ is symmetric.

(b) If $\mathbf{x}(\mathbf{p}, y)$ satisfies WARP, then the 'revealed preferred to' relation, R, has no intransitive cycles. (By definition, $\mathbf{x}^1 R \mathbf{x}^2$ if and only if $\mathbf{x}^1$ is revealed preferred to $\mathbf{x}^2$.)

(a)

This is a 2-dimension case.

A 2x2 Slutsky matrix is symmetric implies that $S_{12} = S_{21}$

$$S_{12} = \frac{\partial x_1}{\partial p_2} + x_2 \frac{\partial x_1}{\partial y} = \frac{\partial x_2}{\partial p_1} + x_1 \frac{\partial x_2}{\partial y} = S_{21}$$

As $x_2(p_1, p_2, y)$ is homogeneous of degree 0.

$$\Rightarrow \frac{\partial x_2}{\partial p_1} p_1 + \frac{\partial x_2}{\partial p_2} p_2 + \frac{\partial x_2}{\partial y} \cdot y = 0$$

$$\Rightarrow \frac{\partial x_2}{\partial y} = -\frac{1}{y}\left(\frac{\partial x_2}{\partial p_1} \cdot p_1 + \frac{\partial x_2}{\partial p_2} \cdot p_2\right)$$

Substitute this back to S_{21}

$$S_{21} = \frac{1}{y}\left(y \frac{\partial x_2}{\partial p_1} - x_1\left(\frac{\partial x_2}{\partial p_1} \cdot p_1 + \frac{\partial x_2}{\partial p_2} \cdot p_2\right)\right)$$

$$= \frac{1}{y}\left[(y - x_1 p_1)\frac{\partial x_2}{\partial p_1} - \frac{\partial x_2}{\partial p_2} \cdot p_2\right]$$

$$= \frac{1}{y}\left[x_2 p_2 \frac{\partial x_2}{\partial p_1} - x_1 p_2 \frac{\partial x_2}{\partial p_2}\right]$$

Similarly, for x_1, can derive $\frac{\partial x_1}{\partial y} = -\frac{1}{y}\left(\frac{\partial x_1}{\partial p_2} \cdot p_2 + \frac{\partial x_1}{\partial p_1} p_1\right)$

substitute this back to S_{12},

$$S_{12} = \frac{1}{y}\left[p_1 x_1 \frac{\partial x_1}{\partial p_2} - p_1 x_2 \frac{\partial x_1}{\partial p_1}\right]$$

As $x_1 p_1 + x_2 p_2 = M$

differentiate w.r.t. $p_1 \Rightarrow x_1 + p_1 \frac{\partial x_1}{\partial p_1} + p_2 \frac{\partial x_2}{\partial p_1} = 0$ ①

w.r.t. $p_2 \Rightarrow p_1 \frac{\partial x_1}{\partial p_2} + x_2 + p_2 \frac{\partial x_2}{\partial p_2} = 0$ ②

① $\cdot x_2$ − ② $\cdot x_1 \Rightarrow x_2 p_1 \frac{\partial x_1}{\partial p_1} + x_2 p_2 \frac{\partial x_2}{\partial p_1} - p_1 x_1 \frac{\partial x_1}{\partial p_2} - p_2 x_1 \frac{\partial x_2}{\partial p_2} = 0$

This implies that. $-x_2p_1\frac{\partial x_1}{\partial p_1}+p_1x_1\frac{\partial x_1}{\partial p_2} = x_2p_2\frac{\partial x_2}{\partial p_1} - p_2x_1\frac{\partial x_2}{\partial p_2}$

$$\Rightarrow \quad y\cdot S_{12} = y\cdot S_{21}$$

$$\Rightarrow \quad S_{12} = S_{21}.$$

(b) This question is actually saying that WARP is enough. for the revealed preference relationship "R" to have no intransitive cycles when there are only 2 goods.

Define $p^1 = (\frac{p_1^1}{p_2^1}, 1)$, x^1 is chosen under p^1

$p^2 = (\frac{p_1^2}{p_2^2}, 1)$. x^2 is chosen.

$p^3 = (\frac{p_1^3}{p_2^3}, 1)$. x^3 is chosen.

Assume x^1 is R.P. to x^2. x^2 is R.P. to x^3

we want to show x^3 is not R.P. to x^1.

i.e. $p^3x^3 - p^3x^1 < 0$

By assumption. we have
$$p^1x^1 \geq p^1x^2$$
$$p^2x^1 > p^2x^2$$
$$p^2x^2 \geq p^2x^3$$
$$p^3x^2 > p^3x^3$$

Since one of the following statement must be true:

① p^1 lies in between p^2 & $p^3 \Rightarrow p^1 = \alpha p^2 + (1-\alpha)p^3$, $\exists i$

② p^2 ... p^1 & $p^3 \Rightarrow p^2 = \alpha p^1 + (1-\alpha)p^3$. $\exists i$

③ p^3 .. p^1 & $p^2 \Rightarrow p^3 = \alpha p^1 + (1-\alpha)p^2$, $\exists i$

We can consider them one-by-one.

Case 1: $p^1 = \alpha p^2 + (1-\alpha)p^3$

$$p^1(x^1-x^3) = p^1(x^1-x^2) + p^1(x^2-x^3)$$

$$= \underbrace{p^1(x^1-x^2)}_{\geq 0} + \underbrace{\alpha p^2(x^2-x^3)}_{\geq 0} + (1-\alpha)\underbrace{p^3(x^2-x^3)}_{>0}$$

$$=$$

$$>0$$

This implies x^1 is revealed preferred to x^3.

By WARP, we have $\underbrace{p^3 x^1 - p^3 x^3 > 0}_{\text{this is what we want to show.}}$

Case 2: $p^2 = \alpha p^1 + (1-\alpha)p^3$

As $p^2(x^1-x^3) = \underbrace{p^2(x^1-x^2)}_{>0} + \underbrace{p^2(x^2-x^3)}_{>0} > 0$

$$p^2(x^1-x^3) = \alpha p^1(x^1-x^3) + (1-\alpha)p^3(x^1-x^3) > 0$$

Then we have to discuss those two terms here

they, at least one of them, have to be positive

$$\left.\begin{array}{l} \text{if } p^1(x^1-x^3) > 0, \text{ By WARP, } p^3(x^1-x^3) > 0 \\ \text{if } p^3(x^1-x^3) > 0, \text{ we don't care } p^1(x^1-x^3) \end{array}\right\} p^3(x^1-x^3) > 0$$

this is what we need.

So in this case, we always have $p^3(x^1-x^3) > 0$

Case 3: $p^3 = \alpha p^1 + (1-\alpha)p^2$

$$p^3(x^1-x^3) = p^3(x^1-x^2) + p^3(x^2-x^3)$$

$$= \underbrace{\alpha p^1(x^1-x^2)}_{\geq 0} + (1-\alpha)\underbrace{p^2(x^1-x^2)}_{>0} + \underbrace{p^3(x^2-x^3)}_{>0} > 0$$

Proof finished!

JR 2.10

2.10 Hicks (1956) offered the following example to demonstrate how WARP can fail to result in transitive revealed preferences when there are more than two goods. The consumer chooses bundle x^i at prices p^i, $i = 0, 1, 2$, where

$$p^0 = \begin{pmatrix}1\\1\\2\end{pmatrix} \quad x^0 = \begin{pmatrix}5\\19\\9\end{pmatrix}$$

$$p^1 = \begin{pmatrix}1\\1\\1\end{pmatrix} \quad x^1 = \begin{pmatrix}12\\12\\12\end{pmatrix}$$

$$p^2 = \begin{pmatrix}1\\2\\1\end{pmatrix} \quad x^2 = \begin{pmatrix}27\\11\\1\end{pmatrix}.$$

(a) Show that these data satisfy WARP. Do it by considering all possible pairwise comparisons of the bundles and showing that in each case, one bundle in the pair is revealed preferred to the other.

(b) Find the intransitivity in the revealed preferences.

How WARP failed to result in transitive revealed preferences when there are more than two goods.

(a) Show WARP : $p^0x^0 \geq p^0x^1$ $p^1x^0 \leq p^1x^1$

(x^0, x^1) $p^0x^0 = 5 + 19 + 18 = 42.$

$p^1x^1 = 12 + 12 + 12 = 36$

$p^0x^1 = 12 + 12 + 24 = 48$

$p^1x^0 = 5 + 19 + 9 = 33.$

Since $p^1x^1 = 36 > p^1x^0 = 33$ x^1 is R.P to x^0

$p^0x^0 = 42 < p^0x^1 = 48$ WARP satisfied for (x^0, x^1)

(x^1, x^2) $p^1x^1 = 36$ $p^2x^2 = 27 + 22 + 1 = 50$

$p^2x^1 = 48$ $p^1x^2 = 40$

Since $p^2x^2 = 50 > p^2x^1 = 48$ x^2 is R.P. to x^1

$p^1x^2 = 40 > p^1x^1 = 36$ WARP satisfied for (x^1, x^2)

(x^0, x^2) $p^0x^0 = 42$ $p^2x^2 = 50$

$p^0x^2 = 40$ $p^2x^0 = 52$

$p^0x^0 = 42 > p^0x^2 = 40$ x^0 is R.P. to x^2

$p^2x^2 = 50 < p^2x^0 = 52$ WARP is satisfied ! for (x^0, x^2)

(b) x^0 is R.P to x^2 , x^2 is R.P. to x^1. x^1. is R.P. to x^0

Intransitive !

2022 Fall Midterm 2. Q4

4.[**15**] The weak axiom of reveled preference (WARP) implies that the demand relations $x_i = \bar{x}_i^M(p_1 \ldots, p_n, M)$ are single valued, i.e., for any price-income vector $(\mathbf{P}, M)$ the consumer chooses a single point of consumption. Prove this result.

Suppose $\exists (P, M)$, such that, at least two bundles are chosen under (P, M)

Assume x^1 is one of the bundle chosen under (p^1, M^1)
x^2 is one of the bundle chosen under (p^2, M^2)
where $p^1 = p^2 = p$, $M^1 = M^2 = M$. $x^1 \neq x^2$
and $p^1 x^1 = M = p^2 x^2$

This implies that, when x^1 is chosen under p^1, x^2 is also affordable.
x^1 is R.P. to x^2
By WARP $p^2 x^2 < p^2 x^1 = p^1 x^1$, impossible

$\Rightarrow x^1 = x^2$.